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The influence of entangled photons on distant persistent currents

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Abstract
The correlations of currents flowing in mesoscopic rings induced by both classical and quantum correlations of photons have been considered. The system of two and three rings in the presence of three types of non-classical radiation, factorizable (uncorrelated), separable (classically correlated) and entangled (quantum mechanically correlated), has been studied. The results show that entangled photons can produce entangled electrons.

1. Introduction

Persistent currents are direct currents driven by static magnetic flux in metallic rings. They are the subject of intensive theoretical and experimental research because there is still disagreement between theory and experiment [1–11]. In either single [12] or multiple [10] metallic ring experiments the measured currents for the case of magnetostatic flux were much larger than accounted for by the theory. Persistent currents in ballistic semiconducting single rings [13] have been also observed with the magnitude expected by the theory. Moreover, the sign of the current still requires an explanation (for a review see e.g. [1]). The next step is to replace the magnetostatic flux with ac magnetic flux (electromagnetic field). The interaction of various mesoscopic systems with electromagnetic fields has been studied in [14, 15].

In the last 20 years the subject of quantum optics has been studied theoretically and experimentally in non-classical electromagnetic fields which are carefully prepared in a particular quantum state. The general idea is to have a fully quantum mechanical system with both the device and the electromagnetic field in the quantum regime. In this case the quantum noise of the electromagnetic field is known and its effect on the currents in the mesoscopic device can be calculated. The interaction of Josephson mesoscopic devices with non-classical electromagnetic fields has been studied in [16, 17]. Experimental work with non-classical fields involving other mesoscopic devices (with Josephson junctions) has recently been reported in [18].

In a previous paper [19], we investigated a different aspect of the persistent currents in mesoscopic rings. In addition to the magnetostatic flux we considered non-classical

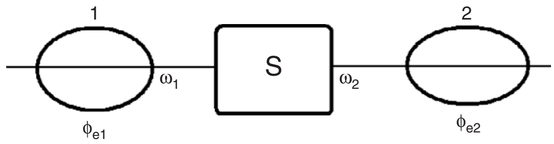

Figure 1. Two mesoscopic rings 1 and 2 are threaded by the magnetostatic fluxes ϕ_{e1} and ϕ_{e2}, correspondingly. An entangled two-mode electromagnetic field is produced by the source S. The first mode of photons with frequency ω_1 interacts with ring 1; and the second mode of photons with frequency ω_2 interacts with ring 2.

electromagnetic fields carefully prepared in a particular quantum state, and studied their effect on the currents. The non-classical electromagnetic fields are of course a small perturbation to the magnetostatic flux; but this perturbation leads to interesting quantum effects that could be observed. In this general context we go further in this paper and study the effect of entangled two-mode electromagnetic fields on persistent currents in two distant mesoscopic rings. We consider two mesoscopic rings which we refer to as 1 ('Alice') and 2 ('Bob'). The static magnetic fluxes ϕ_{e1} and ϕ_{e2} thread the rings 1 and 2, correspondingly. A source S produces entangled two-mode non-classical photons with frequencies ω_1 and ω_2. The photons with frequency ω_1 interact with ring 1, and the photons with frequency ω_2 interact with ring 2 (figure 1).

The purpose of the paper is to show that the entangled photons produce entangled persistent currents in the two rings. We show that classically and quantum mechanically correlated (entangled) photons induce different correlations on distant persistent currents. In particular we show that the correlations depend in a non-trivial way on various parameters of the system such as ring radius, classical magnetic flux and the type of ring. Such information is potentially useful for an experiment in which the current correlations could be measured, and hence this work introduces the possibility of using the mesoscopic rings in the context of quantum communications (Alice and Bob use mesoscopic ring technology).

The properties of the two-mode entangled systems are relatively well understood while multi-mode entangled systems are the subject of current research. Motivated by this we also study the correlations between currents flowing in three rings, each of them threaded by one of the modes. We calculate various quantities which show that all currents are correlated with each other.

In this paper we study fully quantum mechanical systems comprised of mesoscopic devices coupled to the non-classical electromagnetic field. Such devices can be useful for quantum technologies and quantum information processing. The emphasis in these studies is on the properties which cannot be understood classically. The purpose of these considerations is to present some interdisciplinary research which exploits the quantum nature of electromagnetic fields in order to control the behaviour of mesoscopic quantum devices.

2. A single mesoscopic ring

In order to establish the notation and explain the approximations, we briefly review in this section the behaviour of a single mesoscopic ring in the presence of both static magnetic flux and non-classical electromagnetic fields [19]. In this paper we do not address the question of the persistent current amplitude. To perform our model calculations we assume, for simplicity, a 1D mesoscopic ring and calculate the current in the single electron picture neglecting the effect of disorder. However, as stated in the introduction, at least in the case of magnetostatic flux, the currents measured in multichannel diffusive rings are larger than theoretically predicted. Assuming that when the magnetostatic flux is replaced by electromagnetic fields this is also the case, our simple model underestimates the current.

2.1. A single ring threaded by static magnetic flux

Mesoscopic rings in the presence of static magnetic flux ϕ_e exhibit periodic persistent currents which can be paramagnetic (such rings we call p-rings) or diamagnetic (d-rings) at small ϕ_e. The current in p-rings (d-rings) can be modelled as a current in the one-dimensional loop carrying an even (odd) number of electrons respectively. It is interesting to note that rings of single-walled carbon nanotubes behave exactly like real one-dimensional ballistic systems [20].

Let us consider a p-ring of a circumference l_x with M_e electrons threaded by the classical magnetic flux ϕ_e. Persistent current running at temperature T in the ring is given by [21]

$$I(\phi_e/\phi_0, T) = I_0 \sum_{n=1}^{\infty} A_n(T) \sin\left(\frac{2\pi n\phi_e}{\phi_0}\right) \tag{1}$$

with

$$A_n(T) = \frac{4T}{\pi T^*} \frac{\exp(-nT/T^*)}{1 - \exp(-2nT/T^*)} \cos(nk_F l_x) \tag{2}$$

where the flux quantum $\phi_0 = h/e$. The amplitude I_0 of the current is given by

$$I_0 := heM_e/(2l_x^2 m_e) \tag{3}$$

where m_e is an electron mass. The characteristic temperature is given by the relation $k_B T^* = \Delta_F/2\pi^2$ where k_B is the Boltzmann constant, $\Delta_F = h^2 k_F/2\pi m_e l_x$ is the energy gap at the Fermi surface and k_F is the Fermi wavevector.

The current equation (1) is a periodic function of ϕ_e with a period ϕ_0. The characteristics of the current flowing in the d-ring can be obtained by a shift $\phi_e \to \phi_e + \phi_0/2$ in (1).

In the following we limit ourselves to $T = 0$ K when the quantum effects are best visible.

2.2. A single ring threaded by magnetostatic flux and interacting with non-classical microwaves

Quantized electromagnetic fields with frequency ω are described by the vector potential A and the electric field E as dual quantum variables. Below we use units where $\hbar = c = k_B = 1$. Integration of these variables along the circumference of the ring gives the flux $\phi = \oint A\, dl$ and the electromotive force $V_{EM} = \oint E\, dl$. They are not local quantities, but in the case of rings with diameter which is much smaller than the wavelength of the microwaves, we can treat them as dual quantum variables. The flux operator evolves in time as

$$\hat{\phi}(t) = \frac{\sigma}{\sqrt{2}}[\exp(i\omega t)a^\dagger + \exp(-i\omega t)a] \tag{4}$$

where σ is proportional to the area of the ring. This is in the external field approximation where the back-reaction (electromagnetic fields produced by the currents in the ring) is neglected. In other words equation (4) is derived using the free electromagnetic field Hamiltonian

$$H = \omega(a^\dagger a + \tfrac{1}{2}) \tag{5}$$

which does not include photon–ring interaction terms. Such terms will produce corrections to equation (4).

With these approximations the total flux operator (divided over the flux quantum) is

$$\hat{x} = \frac{\phi_\mathrm{e}}{\phi_0} + \frac{\hat{\phi}}{\phi_0} \equiv \lambda + \hat{x}_q, \tag{6}$$

where $\lambda = \phi_\mathrm{e}/\phi_0$ and $\hat{x}_q = \hat{\phi}/\phi_0$.

Another approximation is related to the use of the equilibrium formula given by equation (1). At very high frequencies [22] there are various non-linear effects which we neglect. This is a reasonable approximation when $\omega < \Delta_\mathrm{F}$. A mesoscopic ring with small enough circumference has Δ_F of a few kelvin. Frequencies of the order 10 GHz, used in this paper, correspond to 0.1 K; so this inequality is satisfied. For these conditions the current remains in equilibrium and the effect of radiation can be studied within an adiabatic approximation. For certain values of ϕ ($\phi = k\phi_0$ for p-rings and $\phi = (k+1/2)\phi_0$ for d-rings and integral k) we have a degeneracy of the energy levels. At, and close to, these points the 'adiabatic approximation' is less accurate. However, the elastic (at the degeneracy points) and quasi-elastic (close to the degeneracy points) scattering between these levels does not significantly kill quantum coherence.

Consequently the current (renormalized with division by I_0) is also an operator given by

$$\hat{I}(\hat{x}) = \sum_{n=1}^{\infty} A_n \sin(2\pi n(\lambda + \hat{x}_q)) = \sum_{n=1}^{\infty} \frac{A_n}{2\mathrm{i}} \left[\exp(\mathrm{i}2\pi n\lambda) D(\zeta_n) - \exp(-\mathrm{i}2\pi n\lambda) D(-\zeta_n)\right] \tag{7}$$

where $\zeta_n = n\xi \exp(\mathrm{i}\omega t)$, $\xi = \sqrt{2}\pi\sigma/\phi_0$ and

$$D(\zeta_n) = \exp(\zeta_n a^\dagger - \zeta_n^* a) \tag{8}$$

is a displacement operator. We repeat again that this formula is derived under the equilibrium assumption and it is used here approximately for non-classical electromagnetic fields with small amplitude in comparison to the magnetostatic flux (all our examples have a few photons, for example in equations (14), (16) below); and also for frequencies which are small in comparison to Δ_F.

The expectation values for the current are calculated by taking the trace of the current operator with the density matrix ρ describing the non-classical electromagnetic field:

$$\langle \hat{I}(\hat{x}) \rangle = \sum_n \frac{A_n}{2\mathrm{i}} \left[\exp(\mathrm{i}2\pi n\lambda) W(\zeta_n) - \exp(-\mathrm{i}2\pi n\lambda) W(-\zeta_n)\right], \tag{9}$$

where

$$W(\zeta_n) \equiv \mathrm{Tr}[\rho \exp(\mathrm{i}2\pi n\hat{x}_q)] = \mathrm{Tr}[\rho D(\zeta_n)] \tag{10}$$

are the so-called Weyl functions.

In [19] we have considered various density matrices for the non-classical electromagnetic fields and we calculated the corresponding persistent currents.

3. Two rings with correlated persistent currents

We consider two mesoscopic rings far from each other (figure 1). The static magnetic fluxes $\phi_{\mathrm{e}1}$ and $\phi_{\mathrm{e}2}$ thread the rings 1 and 2, correspondingly. A source S produces two-mode non-classical photons described by the density matrix ρ. The first mode described by the density matrix $\rho_1 = \mathrm{Tr}_2\,\rho$ has frequency ω_1 and interacts with the first ring; and the second mode described by the density matrix $\rho_2 = \mathrm{Tr}_1\,\rho$ has frequency ω_2 and interacts with the second ring. We calculate the quantities

$$\begin{aligned} &\langle \hat{I}_1 \rangle = \mathrm{Tr}(\hat{I}_1 \rho_1); \qquad \langle \hat{I}_2 \rangle = \mathrm{Tr}(\hat{I}_2 \rho_2) \\ &\langle \hat{I}_1 \hat{I}_2 \rangle = \mathrm{Tr}(\hat{I}_1 \hat{I}_2 \rho) \end{aligned} \tag{11}$$

and look for possible correlations of the currents resulting from the correlations of photons.

The density matrix ρ is factorizable when $\rho_{\text{fact}} = \rho_1 \otimes \rho_2$. In this case the two photon modes are uncorrelated. The density matrix ρ is separable if it can be written as

$$\rho_{\text{sep}} = \sum_i p_i \rho_{1i} \otimes \rho_{2\text{i}} \tag{12}$$

where p_i are probabilities. In this case the two photon modes are classically correlated. In all other cases the density matrix ρ is entangled [23] and the photon modes are quantum mechanically correlated.

For factorizable photon density matrices $\langle \hat{I}_1 \hat{I}_2 \rangle = \langle \hat{I}_1 \rangle \langle \hat{I}_2 \rangle$ and the currents in the two rings are uncorrelated. For separable and entangled photon density matrices the currents are correlated and we define the quantity δ as a measure of the correlation of the currents

$$\delta = \langle \hat{I}_2(\hat{x}) \hat{I}_1(\hat{x}) \rangle - \langle \hat{I}_2(\hat{x}) \rangle \langle \hat{I}_1(\hat{x}) \rangle. \tag{13}$$

Clearly in the factorizable case $\delta_{\text{fact}} = 0$.

Since each current loop is equivalent to the magnetic moment the quantity δ is simply the spatial correlation function of the orbital magnetic moments in some analogy to the correlation functions considered in the context of spin systems.

As a first example we consider the photon density matrix

$$\rho_{\text{sep}} = \tfrac{1}{2}(|01\rangle\langle 01| + |10\rangle\langle 10|) \tag{14}$$

where $|01\rangle$ and $|10\rangle$ are the two mode number eigenstates. As a second example we consider the maximally entangled state $|s\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$. The corresponding density matrix is

$$\rho_{\text{ent}} = \rho_{\text{sep}} + \rho_{\text{cross}} \tag{15}$$

$$\rho_{\text{cross}} = \tfrac{1}{2}(|01\rangle\langle 10| + |10\rangle\langle 01|). \tag{16}$$

We note that ρ_{cross} is not a density matrix.

For the classically correlated photons in the state ρ_{sep}

$$\begin{aligned} \delta_{\text{sep}} = \tfrac{1}{2} \sum_{nm} & A_n A_m \sin(2\pi n \lambda_1) \sin(2\pi m \lambda_2) \\ & \times \exp[-(n^2 + m^2)\xi^2/2](2 - (n^2 + m^2)\xi^2) - \langle I_1(\lambda_1) \rangle \langle I_2(\lambda_2) \rangle \end{aligned} \tag{17}$$

where

$$\langle I_j \rangle = \tfrac{1}{2} \sum_n A_n \sin(2\pi n \lambda_j) \exp(-n^2 \xi^2/2)(2 - n^2 \xi^2) \tag{18}$$

for $j = 1, 2$.

For the entangled photons the qualitative difference appears. The correlation function is now time dependent. It consists of two parts, one of them corresponding to ρ_{sep} and the other one to ρ_{cross} in the total density operator ρ_{ent}

$$\delta_{\text{ent}} = \delta_{\text{sep}} + \delta_{\text{cross}} \tag{19}$$

with the 'cross' term

$$\delta_{\text{cross}} = \sum_{nm} A_n A_m \exp(-(n^2 + m^2)\xi^2/2) nm \xi^2 \cos(2\pi n \lambda_1) \cos(2\pi m \lambda_2) \cos[(\omega_1 - \omega_2)t]. \tag{20}$$

We see that δ_{ent} possess a term which oscillates in time with frequency $(\omega_1 - \omega_2)$. If $\omega_1 = \omega_2$ the correlation function δ_{ent} differs from δ_{sep} by a time-independent value.

It follows from equations (17) and (19) that contrary to typical spin systems the correlation represented by δ does not depend on the distance between the rings [24]. This is because we

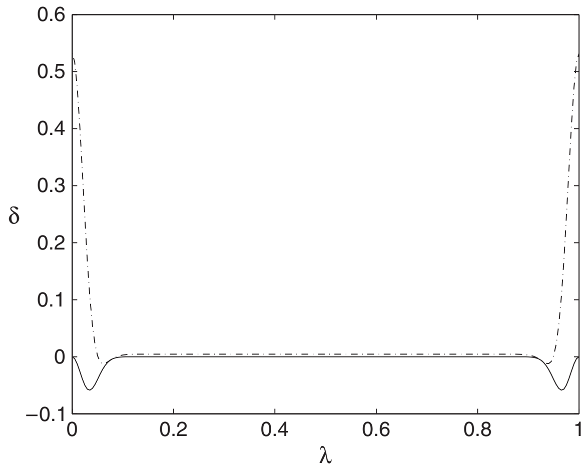

Figure 2. The correlation functions δ_{sep} of equation (17) (solid lines) and δ_{ent} of equation (19) (dashed lines) for a system of two mesoscopic p-rings. δ_{sep} does not depend on time. δ_{ent} is a sinusoidal function of time (equation (20)) and the curve shown is for $t = 0$.

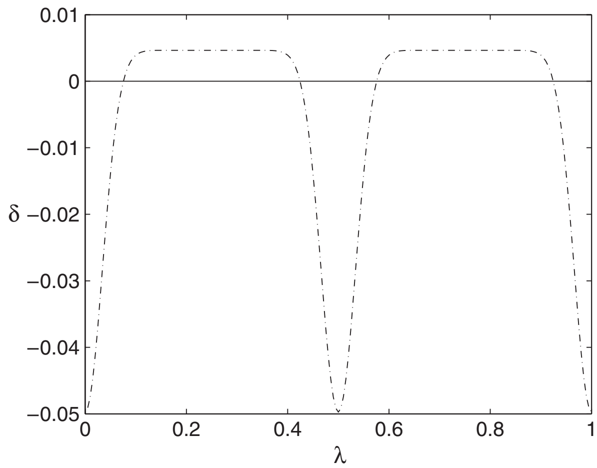

Figure 3. The correlation functions δ_{sep} of equation (17) (solid lines) and δ_{ent} of equation (19) (dashed lines) for a system comprised of a p-ring and a d-ring. δ_{sep} does not depend on time. δ_{ent} is a sinusoidal function of time (equation (20)) and the curve shown is for $t = 0$.

neglect the effect of noise and assume that the state of photons remains pure and hence the correlations of photons both classical and quantum are distance independent.

In figures 2–5 we present numerical results for δ. We have chosen equal static magnetic fluxes in the two rings $\lambda \equiv \lambda_1 = \lambda_2$ and $\sigma = 1$. The value of $\omega_1 - \omega_2$ is 10^{-5} eV (13.5 GHz). The values of ω_i, $(i = 1, 2)$ are such that the 'equilibrium conditions' ($\omega_i < \Delta_{\mathrm{F}}$) are satisfied. It can easily be done with rings of $l_x \sim 1\ \mu$m [2].

In the following we discuss two cases. The first is when there are two p-rings and the second is when one of the rings is a p-ring while the other is a d-ring. The results for the system of two d-rings can be deduced by shifting $\lambda \to \lambda + 1/2$ in the figures corresponding to the case of two p-rings.

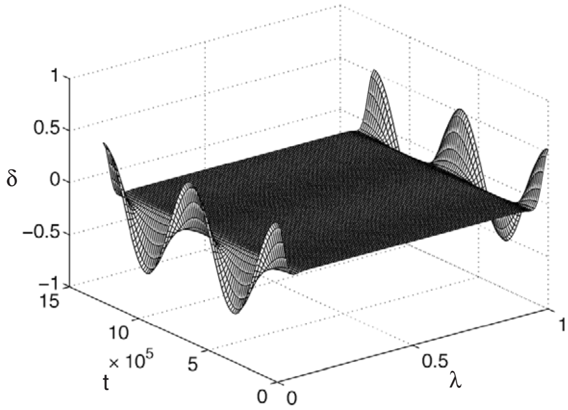

Figure 4. The correlation function δ_{ent} of equation (19) as a function of time t and the classical magnetostatic flux λ in a system of two p-rings.

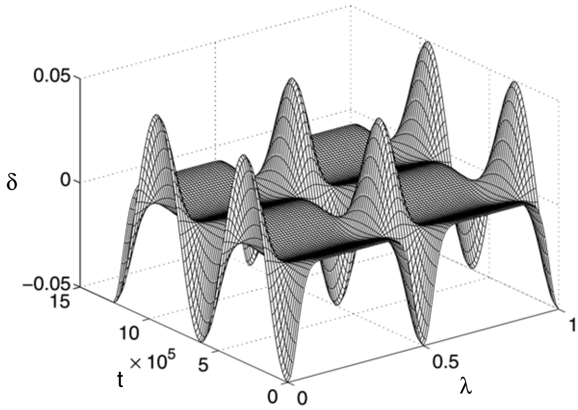

Figure 5. The correlation function δ_{ent} of equation (19) as a function of time t and the classical magnetostatic flux λ in the system of p–d rings.

We see that the quantum correlations carried by ρ_{ent} influence the correlation function much more strongly than the classical ones carried by ρ_{sep} which is almost vanishing in the p–d case.

There is one order of magnitude difference in the amplitude of δ. We also notice that the amplitude of δ_{ent} for rings of the same type i.e. p–p or d–d is again an order of magnitude larger than in the p–d case.

The difference in the correlation functions δ_{sep} and δ_{ent} depends on the magnitude of the classical flux λ. For example for the system of two p-rings (figure 2) it is the largest for λ near 0 (modulo 1). In the system of two d-rings it happens for λ near $1/2$ (this can be seen from figure 2 with $\lambda \to \lambda + 1/2$).

The detailed analysis shows that δ_{ent} is substantial only in the regions where the currents $I_i(\lambda_i)$ exhibit a steep change. The correlation function is much larger in the case of the same rings (p–p or d–d) because the changes of $I_i(\lambda_i)$ are then 'in phase'. We also checked that the correlations between currents are the largest in the case of rings of equal circumference, i.e. $l_{x1} = l_{x2}$. For example if one considers two rings such that $l_{x1} \approx 3l_{x2}$ the resulting amplitude of the correlation function is lowered almost twice.

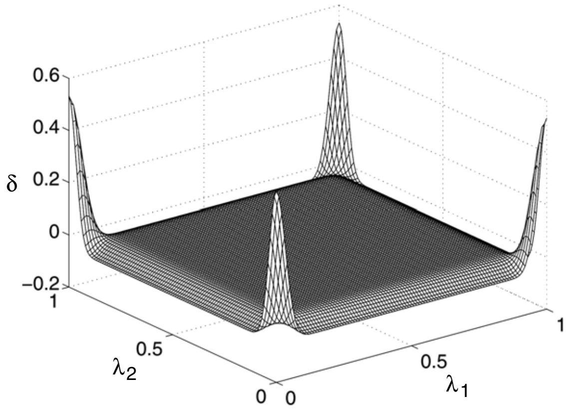

Figure 6. The correlation function δ_{ent} as a function of the classical magnetostatic fluxes λ_1 and λ_2 in a system of two p-rings. The result is a sinusoidal function of time (equation (20)) and the curve shown is for $t = 0$.

Besides, there is a substantial difference with allows to distinguish the correlation of the current inherited from the classical correlation of non-classical electromagnetic fields from the quantum correlations since the latter are time dependent.

Let us notice that the presence of the static flux λ in both rings is not necessary for the existence of the correlation between them (equations (17)–(20)). If $\lambda_1 = \lambda_2 = 0$ the correlation $\delta_{\text{sep}} = 0$ and the *expectation* values of currents in both rings vanish but there is still a 'zero-flux' correlation δ_{ent} which is finite provided that the state of the two photons is entangled. The current fluctuations do inherit the quantum correlations of the photonic field.

Further we study the correlations in the system with $\lambda_1 \neq \lambda_2$. The numerical results for the p–p configuration are given in figure 6. We see that the amplitude of the correlations between the rings depends strongly on the values of the fluxes threading the rings. If at least one of the classical fluxes λ_i is fixed at some value which is far from $\lambda = 0$ (modulo 1), the correlation of the currents is small. This suggests that the quantum correlations of currents inherited from the entanglement of photons can be either enhanced or suppressed by the proper choice of parameters of the system. A similar effect appears for other configurations. For the system of two d-rings the setting $\lambda_1 \approx \lambda_2 \approx 1/2$ leads to the maximal values of δ_{ent}. At the same time setting up $\lambda_1 \approx 1/2$ and $\lambda_2 \approx 0$ (modulo 1) leads to the maximal correlations in the mixed d–p configuration. These effects are important for any attempt of experimental verifications of the presented theoretical considerations.

4. Three-mode entanglement

It is known that three-mode entanglement is not a trivial generalization of two-mode entanglement (e.g. [25]). For example, one approach to entanglement is with entropies. The strong subadditivity property in this context provides a deeper insight into the difference between two-mode and three-mode entanglement.

In this section we consider three rings far from each other. Each ring is threaded by a static flux. The source S produces a three-mode electromagnetic field described by a density matrix ρ. We define the quantities

$$\begin{array}{lll} \rho_1 = \mathrm{Tr}_{23}\,\rho; & \rho_2 = \mathrm{Tr}_{13}\,\rho; & \rho_3 = \mathrm{Tr}_{12}\,\rho \\ \rho_{12} = \mathrm{Tr}_3\,\rho; & \rho_{13} = \mathrm{Tr}_2\,\rho; & \rho_{23} = \mathrm{Tr}_1\,\rho. \end{array} \tag{21}$$

The first mode described by ρ_1, having frequency ω_1, interacts with the first ring; similarly with the other two modes which interact with the second and third ring, respectively. We calculate the quantities

$$\begin{aligned} &\langle \hat{I}_i \rangle = \mathrm{Tr}(\hat{I}_i \rho_i); \qquad \langle \hat{I}_i \hat{I}_j \rangle = \mathrm{Tr}(\hat{I}_i \hat{I}_j \rho_{ij}) \\ &\langle \hat{I}_1 \hat{I}_2 \hat{I}_3 \rangle = \mathrm{Tr}(\hat{I}_1 \hat{I}_2 \hat{I}_3 \rho) \end{aligned} \tag{22}$$

and the differences

$$\delta_{ij} = \langle \hat{I}_i \hat{I}_j \rangle - \langle \hat{I}_i \rangle \langle \hat{I}_j \rangle \tag{23}$$

$$\delta_{123} = \langle \hat{I}_1 \hat{I}_2 \hat{I}_3 \rangle - \langle \hat{I}_1 \rangle \langle \hat{I}_2 \rangle \langle \hat{I}_3 \rangle \tag{24}$$

$$\delta_{12-3} = \langle \hat{I}_1 \hat{I}_2 \hat{I}_3 \rangle - \langle \hat{I}_1 \hat{I}_2 \rangle \langle \hat{I}_3 \rangle . \tag{25}$$

As an example we consider the photon state

$$|s\rangle = 2^{-1/2}[|000\rangle + |123\rangle] \tag{26}$$

expressed as a combination of the three mode number eigenstates. Its density matrix is

$$\rho = \tfrac{1}{2}(|000\rangle\langle 000| + |123\rangle\langle 123| + |000\rangle\langle 123| + |123\rangle\langle 000|), \tag{27}$$

where the first two terms are separable and the other two are cross terms. We also define

$$\rho_{ij} = \tfrac{1}{2}\,(|00\rangle\langle 00| + |ij\rangle\langle ij|) \tag{28}$$

$$\rho_i = \tfrac{1}{2}\,(|0\rangle\langle 0| + |i\rangle\langle i|)\,. \tag{29}$$

Although the state ρ is entangled, the states ρ_{ij} are separable. Correspondingly we calculate

$$\begin{aligned} \langle I_i \rangle &= \tfrac{1}{2} \sum_{n=1}^{\infty} A_n \sin(2\pi n \lambda_i) \exp(-n^2 \xi^2/2)[1 + L_i(n^2\xi^2)] \\ \langle I_i I_j \rangle &= \tfrac{1}{2} \sum_{m,n=1}^{\infty} A_m A_n \sin(2\pi n \lambda_i) \sin(2\pi m \lambda_i) \\ &\quad \times \exp[-(m^2+n^2)\xi^2/2][1 + L_i(m^2\xi^2) L_j(n^2\xi^2)] \\ \langle I_1 I_2 I_3 \rangle &= \tfrac{1}{2} \sum_{m,n,j=1}^{\infty} A_m A_n A_j \sin(2\pi m \lambda_1) \sin(2\pi n \lambda_2) \sin(2\pi j \lambda_3) \\ &\quad \times \exp[-(m^2+n^2+j^2)\xi^2/2][1 + L_1(m^2\xi^2) L_2(n^2\xi^2) L_3(j^2\xi^2)] \\ &\quad - \frac{1}{2\times 3^{1/2}} \sum_{m,n,j=1}^{\infty} A_m A_n A_j \cos(2\pi m \lambda_1) \sin(2\pi n \lambda_2) \cos(2\pi j \lambda_3) \\ &\quad \times \cos[(\omega_1 + 2\omega_2 + 3\omega_3)t] m n^2 j^3 \xi^6 \exp[-(m^2+n^2+j^2)\xi^2/2] \end{aligned} \tag{30}$$

where L_i are the Laguerre polynomials. We see that only $\langle I_1 I_2 I_3 \rangle$ is time dependent. We show the numerical results for δ_{12}, δ_{123} and δ_{12-3} against λ in figures 7–11. Here we let $\omega_1 = 10^{-5}$ (13.5 GHz), $\omega_2 = 2\omega_1$, $\omega_3 = 3\omega_1$ and $\lambda_1 = \lambda_2 = \lambda_3 = \lambda$. In figure 7 we plot δ_{12} as a function of λ. Comparing these results with the solid line in figure 2 we see that the results are similar, in spite of the fact that the states in the two cases are slightly different. In figure 8 we plot δ_{123} for p–p–p rings, as a function of λ and t. It is seen that we get significant time-dependent results only around $\lambda = 0$ (modulo 1). In figure 9 we present similar results for p–d–p rings. In figures 10, 11 we plot δ_{12-3} as a function of λ and t for p–p–p and p–d–p rings, correspondingly.

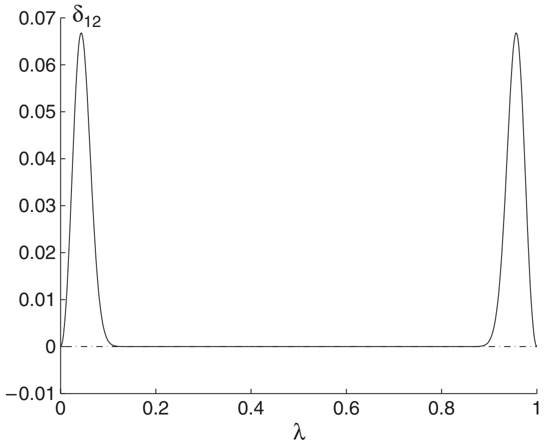

Figure 7. The correlation functions δ_{12} of equation (23) for p–p rings (solid line) and for p–d rings (broken line).

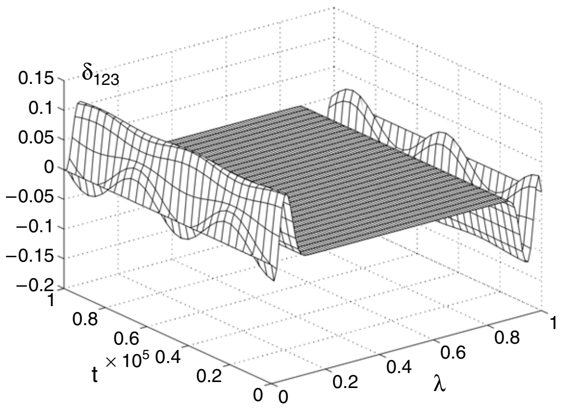

Figure 8. The correlation functions δ_{123} of equation (24) as a function of the classical magnetostatic flux λ and the time t; for p–p–p rings.

5. Discussion and conclusion

We have considered a system of two and three distant mesoscopic rings irradiated with a static magnetic flux and non-classical multi-mode electromagnetic field.

In such a situation persistent currents are given by the expectation values of the current operators with the density operators of the electromagnetic fields. In the case of two rings we compared the influence of different two-photon states on two persistent current loops. We calculated the joint persistent current and quantified the correlations between the two rings with the correlation function δ for three different states of electromagnetic fields: factorizable (uncorrelated), separable (classically correlated) and entangled (quantum mechanically correlated).

Thus we were able to compare and contrast the influence of various (classical and quantum) correlations on two distant persistent currents. The results show that classically (described by ρ_{sep}) and quantum mechanically correlated (described by ρ_{ent}) photons induce different current

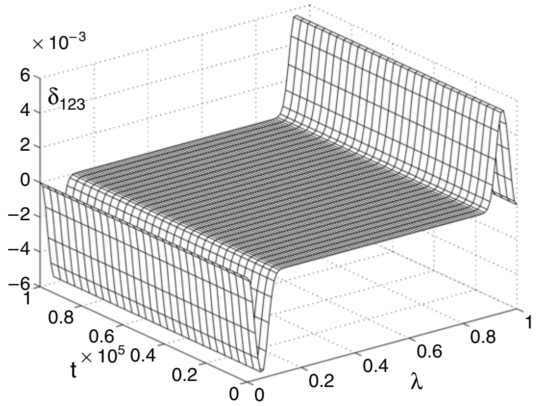

Figure 9. The correlation functions δ_{123} of equation (24) as a function of the classical magnetostatic flux λ and the time t; for p–d–p rings.

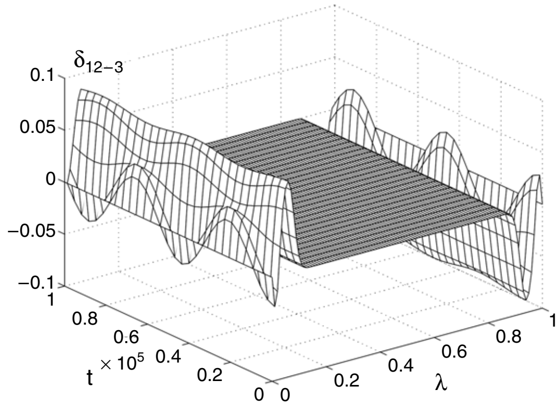

Figure 10. The correlation functions δ_{12-3} of equation (25) as a function of the classical magnetostatic flux λ and the time t; for p–p–p rings.

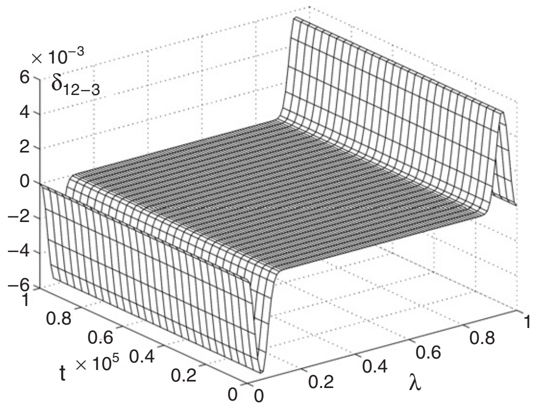

Figure 11. The correlation functions δ_{12-3} of equation (25) as a function of the classical magnetostatic flux λ and the time t; for p–d–p rings.

correlations. We have shown that the effect of ρ_{cross} on δ is substantial and helps us to distinguish the currents driven by separable and entangled photons.

It follows from the presented considerations that the quantum correlations represented by δ_{ent} depend strongly on the parameters of the system such as classical magnetic fluxes λ_i or sizes of the rings. These results should indicate which parameters one should use to observe the phenomenon experimentally.

Further, we considered the system of three rings irradiated by three-mode fields. As an example we chose an entangled three-mode field and showed that such states produce non-trivial effects on the correlations of currents in the rings.

In our model calculations we assumed, for simplicity, one-dimensional rings. In realistic many-channel rings the current increases with the number of channels. The energy gap preserving coherence in such a multi-channel system is still given by energy level separation for motion around the ring [2]. The small and moderate disorder decreases somewhat the current but does not destroy it.

Mesoscopic systems carrying persistent currents appear as a possibly useful tool in experiments measuring the properties of entangled light. The examples presented offer novel applications of mesoscopic rings in quantum communication, for example as a part of a device for an entanglement detection.

Since persistent currents or the equivalent orbital magnetic moments are also the property of the carbon nanotubes our considerations may form another bridge between nanotechnology and quantum information processing.

Recently the system of two separated interference experiments irradiated by two possibly entangled photons has been considered [26]. It has been shown that there is a possibility of obtaining entangled electrons in such a way.

As there is a one to one correspondence between a state of a two-mode or three-mode electromagnetic field and the persistent currents we can state that entangled photons produce entangled currents (entangled electrons). In that sense the quantum mechanical correlations present in non-classical electromagnetic fields are inherited by the currents. An experiment in which the correlation function or its power spectrum could be measured would clearly demonstrate that entangled photons produce entangled electrons.

The work is in the general area of mesoscopic devices applied to quantum information processing. We note that there is a lot of work on entanglement detection [27] which can play a complementary role to our work.

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References

[1] Eckern U and Schwab P 2002 *J. Low Temp. Phys.* **126** 1291
[2] Imry Y 1997 *Introduction to Mesoscopic Physics* (Oxford: Oxford University Press) pp 69–71
[3] Washburn S 1986 *R. A. Webb Adv. Phys.* **35** 375
Aronov A G and Sharvin Y V 1987 *Rev. Mod. Phys.* **59** 755

Pepper M 1988 *Proc. R. Soc.* A **420** 1
[4] Buttiker M, Pretre A and Thomas H 1993 *Phys. Rev. Lett.* **70** 4114
Pieper J B and Price J C 1994 *Phys. Rev. Lett.* **72** 3586
Cohen J and Avishai Y 1995 *J. Phys.: Condens. Matter* **7** 8791
[5] Gefen Y, Imry Y and Azbel M Ya 1984 *Phys. Rev. Lett.* **52** 129
Buttiker M, Imry Y and Azbel M Ya 1984 *Phys. Rev.* A **30** 1982
[6] Reulet B, Bouchiat H and Mailly D 1995 *Europhys. Lett.* **31** 305
Deblock R, Noat Y, Bouchiat H, Reulet B and Mailly D 2000 *Phys. Rev. Lett.* **84** 5379
Deblock R, Bel R, Reulet B, Bouchiat H and Mailly D 2002 *Phys. Rev. Lett.* **89** 206803
[7] Kopietz P 1992 *Phys. Rev.* B **46** 2280
Volker A and Kopietz P 1997 *Z. Phys.* B **102** 545
Volker A and Kopietz P 2002 *Phys. Rev.* B **65** 045112
[8] Jariwala E M Q, Mohanty P, Ketchen M B and Webb R A 2001 *Phys. Rev. Lett.* **86** 1594
Mohanty P 1999 *Ann. Phys., Lpz.* **8** 549
[9] Büttiker M, Imry Y and Landauer R 1983 *Phys. Lett.* A **96** 365
[10] Levy L P, Dolan G, Dunsmuir J and Bouchiat H 1990 *Phys. Rev. Lett.* **64** 2074
Levy L P 1991 *Physica* B **169** 245
[11] Czajka K, Maśka M, Mierzejewski M and Śledź Ż 2005 *Phys. Rev.* B at press
[12] Chandrasekhar V *et al* 1991 *Phys. Rev. Lett.* **67** 3578
[13] Mailly D *et al* 1990 *Phys. Rev. Lett.* **70** 2020
[14] Deblock R, Noat Y, Reulet B, Bouchiat H and Mailly D 2002 *Phys. Rev.* B **65** 075301
[15] Kravtsov V E and Altshuler B L 2000 *Phys. Rev. Lett.* **84** 3394
Wang X B and Kravtsov V E 2001 *Phys. Rev.* B **64** 033313
Basko D M and Kravtsov V E 2004 *Phys. Rev. Lett.* **93** 056804
Basko D M and Kravtsov V E 2005 *Phys. Rev.* B **71** 085311
[16] Vourdas A 1994 *Phys. Rev.* B **49** 12040
Odintsov A A and Vourdas A 1996 *Europhys. Lett.* **34** 385
Vourdas A 1996 *Z. Phys.* B **100** 455
[17] Kuang L M, Wang Y and Ge M L 1996 *Phys. Rev.* B **53** 11764
Zou J, Shao B and Xing X S 1997 *Phys. Rev.* B **56** 14116
Al-Saidi W and Stroud D 2002 *Phys. Rev.* B **65** 014512
[18] Wallraff A *et al* 2004 *Nature* **431** 162
Blais A *et al* 2004 *Phys. Rev.* A **69** 062320
[19] Dajka J, Szopa M, Vourdas A and Zipper E 2004 *Phys. Rev.* B **69** 045305
[20] Latil S, Roche S and Rubio A 2003 *Phys. Rev.* B **67** 165420
[21] Cheung H F, Gefen Y, Riedel E K and Shih W H 1988 *Phys. Rev.* B **37** 6050
[22] Kravtsov V E and Yudson V I 1993 *Phys. Rev. Lett.* **70** 210
Aronov A G and Kravtsov V E 1993 *Phys. Rev.* B **47** 13409
[23] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)
[24] Greven M, Birgeneau R J and Wiese U-J 1996 *Phys. Rev. Lett.* **77** 1865
Kim Y J, Greven M, Wiese U-J and Birgeneau R J 1998 *Eur. Phys. J.* B. **4** 291
Giovannetti V and Fazio R 2005 *Phys. Rev.* A **71** 032314
[25] Roos Ch F *et al* 2004 *Science* **304** 1478
[26] Tsomokos D I, Chong C C and Vourdas A 2004 *Phys. Rev.* A **69** 013810
Tsomokos D I, Chong C C and Vourdas A 2005 *J. Phys.: Condens. Matter* at press
Paternostro M, Falci G, Kim M and Palma G M 2004 *Phys. Rev.* B **69** 214502
[27] Sancho J M G and Huelga S F 2000 *Phys. Rev.* A **61** 042303
Ekert A K *et al* 2002 *Phys. Rev. Lett.* **88** 217901
Horodecki P and Ekert A 2002 *Phys. Rev. Lett.* **89** 127902
Horodecki P 2003 *Phys. Rev. Lett.* **90** 167901